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Seismic zoning for initial- and total-cost minimization

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SUMMARY

We consider two design criteria to study seismic zoning. In the first, codes require that structures be designed for some specified values. Zoning is then optimal when it minimizes the expected present value of the initial costs of all structures to be built in the region being zoned. In the second criterion, it is designed so that the present value of the total cost is minimized, including initial and maintenance costs as well as losses due to damage and failure. We will call these criteria zoning for the *initial*- and *total-cost minimization*, respectively. It is shown that under certain conditions, the boundaries coincide with isoparametric curves and the problem may be solved in one dimension. We also deal with problems not reducible to a single dimension. Different methods are proposed to solve the various kinds of problems. The work ends with some illustrative examples. Copyright © 2000 John Wiley & Sons, Ltd.

KEY WORDS: zoning; standardization; optimization; seismic zoning; earthquake resistant design; optimal zoning

INTRODUCTION

The geographic variation of design requirements over a region can be represented by isoparametric curves or through zoning. Seismic zoning consists of dividing a region into portions where certain constant seismic-design parameters are specified. By adopting the zoning whose boundaries coincide with jurisdictional limits we avoid two aspects: one is the ambiguities caused by using isoparametric curves, ambiguities that can be important when we use maps of manageable size, and the other is the arbitrariness introduced by interpolation. In addition, the applicability of normative requirements, where the zoning belongs, is simplified. Zoning is therefore often justified. We will consider two design criteria. In the first, each structure is to be designed for not less than certain values of parameters or coefficients specified in a map of isoparametric curves, so when we zone almost all structures are overdesigned, and only initial cost is of concern. The problem consists of finding the interzone boundaries that minimize the waste caused by zoning. We call this criterion *initial-cost minimization*. In the second, constant design coefficients will be

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specified in each zone, for the different types of structures that will be built. These coefficients and the interzone boundaries must minimize the expected present value of the structures' total cost, including initial and maintenance costs as well as losses due to damage caused by earthquakes. We call this criterion *total-cost minimization*. The second criterion is justified in most cases because the monetary quantities involved are usually small enough compared with the society's resources to that they can be assumed to be linearly related to utility.

In a previous paper by Rosenblueth and García-Pérez [1] efficient methods applicable in one dimension were developed. Here, we review the assumptions to reduce the multidimensional problem to a one-dimensional one. Then we extend the optimum zoning methods to problems that must be treated in two dimensions. Initially, we deal with unrestricted boundaries (when zone boundaries do not coincide with well-recognized lines such as jurisdictional boundaries), and later on we release this constraint. All of the above procedures are used for the two aforementioned design criteria.

STATEMENT OF THE PROBLEM

Let x, y denote the coordinates of a point in the region to be zoned and structures type $i = 1, \ldots, I$, and zone $k = 1, \ldots, K$. Also, let $Z = Z(x, y)$ be the vector defining seismicity (in zoning for initial-cost minimization, Z is the corresponding vector of minimum design coefficients specified), let c be the vector of design coefficients adopted, $u = u(c, x, y)$ and $w = w(c, Z, x, y)$ the initial and the expected present value of the total cost of a structure respectively, $\phi = \phi(x,y)$ the expected present value of the number of structures to be built per unit area, and U and W, the expected present value of the initial and the total cost of all structures, respectively. Then $U_{ik} = \iint_k \phi_i u_i \,\mathrm{d}x\,\mathrm{d}y$, $U_k = \sum_{i=1}^{1} U_{ik}$, $U = \sum_{k=1}^{K} U_k$, and similarly for W_{ik} W_k and W. The problem is to minimize U or W.

CONDITIONS FOR REDUCTION TO 1D

Optimum zoning problems can be reduced to a single dimension when certain requirements are met. These requirements are the same for both the design criteria considered here.

We assume initially that structures of a single type will be built in the region to be zoned. Thus, the initial cost u of a structure depends only on the base shear coefficient c for which it is designed and, it is also an increasing function of this coefficient. In zoning for the initial-cost minimization, this is the only unit cost of concern, and we denominate u as w. In the total-cost design, the expected present value v of the losses affecting the structure during an earthquake is also of interest. This expectation will depend on c and also on a functional Z of the site seismicity where the structure will be built, that is, a functional of the demands of the base shear coefficient (let us say, ordinates of the pseudo-acceleration spectrum expressed in terms of the gravity acceleration), so that v is a decreasing function of c and an increasing function of Z. We assume that a map of the region with isoparametric curves Z has been drawn in the design for total-cost minimization, but corresponding in design of initial-cost minimization to constant values of the minimum coefficient for which we should design. We also call it Z, and it would be the design coefficient if it is not zoned. We will call these isoparametric curves *isoseismals*. In both types of design, each curve of the constant Z is also of the constant w. The map will also contain curves with constant

values ϕ of the expected present value of the number of structures that will be built per unit area and per unit time. We will divide the region into zones that we will call k, $k = 1, \ldots, K$, and we will take K as known. Structures in zone k will be designed for a constant design coefficient c_k. We are interested first in computing each c_k that minimizes the expected present value of the cost of all structures in zone k, $W_k = \int_k \phi w(c_k, Z)\,\mathrm{d}A$, where A means area. Then, we are interested in defining the boundaries between zones so that the total cost in the region $W = \sum_{k=1}^{K} W_k$ is minimum.

In the design for initial-cost minimization, c_k is the maximum value of Z in zone k, so that we can write $w(c_k, Z) = w(c_k)$, unit cost that we call u_k; therefore, $U_k = u_k \int_k \phi\,\mathrm{d}A$. In the design for the total-cost, c_k must be computed explicitly and the simplification mentioned in the initial-cost minimization is not allowed.

Under these conditions interzone boundaries coincide with *isoseismals* and the zoning problem can be reduced to a single dimension where it is advisable to take $A_k = A(c_k)$ as the area in which the design coefficient does not exceed c_k, so the area of zone k is $A_k - A_{k-1}$, with $A_0 = 0$ and taking $A(Z)$ as the area in which the parameter defining seismicity does not exceed Z. The problem becomes formally equal to that of catalogue optimization of standardized products defined by a single parameter [2–4], Z being the size or demanded capacity, c_k the kth delivered size, w the unit cost of the product and, if we define $f(Z)$ as the integral of ϕ through the isoseismal Z, f is the frequency of use or the expected present value of the number of products that will be demanded per unit Z.

The reduction to one dimension is also applicable in the case where different types $i = 1, \ldots, I$, of structures will be built, corresponding to each of them a map with curves of Z_i and ϕ_i, provided that two conditions are met: (a) all Z_is increase monotonically with each other, so it suffices to consider any such component to define all of them and (b) the ratios ϕ_i/ϕ_j are constant in the region for all i and j. If only the first condition is met, interzone boundaries will still continue coinciding with isoseismals, but the problem will be treated in multiple dimensions.

When both conditions are met, we can have a single type of structure whose cost is the weighted average of those of the corresponding types that will be built. We then define $\phi = \sum_{i=1}^{I} \phi_i$ and $w = \sum_{i=1}^{1} \phi_i w_i/\phi$. Therefore, the problem can still be treated in one dimension.

When the above condition is satisfied, the normative seismicity is defined totally by a single set of isoparametric curves joining constant values of some design coefficient c. We can call these curves *generalized isoseismals*. They correspond, for example, to a fixed exceedance rate if we assume that arrival times of the significant earthquakes to each site can be idealized as those corresponding to a multiple Poisson process, that is, without memory, or maybe by taking into account variations of the probability distributions of arrival times as a function of the magnitudes and of the occurrence times of earthquakes that have already occurred.

ZONING WITH UNRESTRICTED BOUNDARIES

Initial-cost minimization

When reducible to 1D. In the initial-cost criterion we establish in each zone coefficients equal to the maximum specified when the region is not zoned, then the waste resulting from zoning is big. This case is thoroughly treated in a paper before by Rosenblueth and García-Pérez [1].

When not reducible to 1D. For the sake of simplicity we will expound efficient methods to solve these problems assuming that in the region of interest only two types of structures will be built (for instance, types whose isoparametric curves are not coincident). Generalization to any number of types is immediate, and it is also the same to situations in which the unit cost of each type is an increasing function, not necessarily additive and not only of one coefficient, but of many.

We will consider the amount $\zeta_{kl} = \sum_{i=1}^{2} (u_{ik} - u_{il}) \phi_i$ where k and l identify two adjacent zones. Let us begin with the case when $c_{1k} < c_{1l}$ and $c_{2k} < c_{2l}$. Wherever $\phi_1 + \phi_2 > 0$ we will have $\zeta_{kl} < 0$. With reference to Figure 1, let us say that in order to limit zones k and l, we have postulated the thick continuous line where the inequalities among the design coefficients are complied; otherwise, it is arbitrary. The postulated line goes through variable values of Z_1 and Z_2, so that $Z_{1k} = \max Z_1 = c_{1k}$ and $Z_{2k} = \max Z_2 = c_{2k}$. If we move the boundary to the discontinuous line that coincides segmentally with isoseismals Z_{1k} and Z_{2k}, then the area between this line and the postulated boundary is moved from zone l to k, therefore, the corresponding design coefficients will diminish from c_{il} to c_{ik}. Thus, the total cost of the structures in the region will have changed from U to $U + \int \zeta_{kl}\, \mathrm{d}A$ (except for the trivial case $\phi_1 = \phi_2 = 0$, where the position of the boundary is irrelevant in the area considered), where the integral extends to the area between the two thick lines, and the discontinuous line will be a better solution than the one postulated. Under these circumstances, no matter what boundary we postulate, we will always find a better one, formed by segments of isoseismals of both types of structure. Therefore, the optimum boundaries must be formed by such segments.

Again in reference to Figure 1, let us choose boundaries as those we just described; let us also maintain their portions fixed coincidently with isoseismals of the second type of structures, and let us move the segment of the boundary between k and l that coincides with one isoseismal of the structures type 1, so that we can pass from Z_{1k} to $Z_{1k} + \mathrm{d}Z$, and so from u_{1k} to $u_{1k} + \mathrm{d}u_1$. By doing so, we assign the shaded surface of area $\mathrm{d}A$ to zone k which we took out of zone l. Let us call $k+1$ to l, F_{ik} and $F_{i,k-1}$, respectively, the number of structures type i in zones k and $k-1$. The value $F_{ik} - F_{i,k-1}$ is increased by $\mathrm{d}F_i = f_i\, \mathrm{d}A$ (where now f_i is the integral of ϕ_i through the segment whose position we modified, that is, f_i is the demand per unit change in A), quantity in

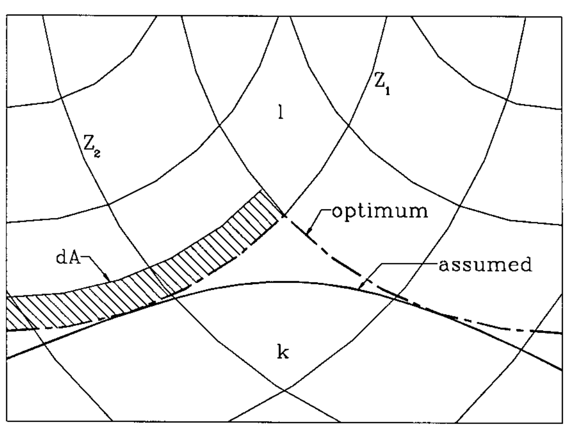

Figure 1. Optimum boundaries for two types of structures, $Z_{ik} < Z_{il}$.

which $F_{1,k+1} - F_{1,k}$ is diminished, while $u_{1,k}$ increases in $\mathrm{d}u_1$, and $u_{2,k}$ is not affected. If the boundary with the discontinuous line were optimum, the contributions of zones k and $k+1$ to the total cost of the region (U) before and after moving the curve u_{1k} would be stationary. Thus, because only u_k and u_{k+1} are modified, if we make $u_k = (F_{1k}u_{1k} + F_{2k}u_{2k})/F_k$, $F_k = F_{1k} + F_{2k}$, the following must be satisfied $(F_k - F_{k-1})u_k + (F_{k+1} - F_k)u_{k+1} = (F_k - F_{k-1} + f_1 \mathrm{d}A_1)(u_k + \mathrm{d}u) + (F_{k+1} - F_k - f_1\, \mathrm{d}A_1)u_{k+1} = 0$, then

$$\frac{\mathrm{d}u}{f_1\,\mathrm{d}A_1} = \frac{u_{k+1} - u_k}{F_k - F_{k-1}} \tag{1}$$

We recognize this equation as the one obtained in the graphical method for one-dimensional problems [1]. Therefore, if the segments of the boundaries coincidently with isoseismals for type 2 were correct, we can set up the following procedure:

1. Choose tentative values of u_{1k}.
2. Use any of the methods for one dimension to find the optimum u_{2k} corresponding to u_{1k}, and compute U.
3. Repeat with another set of tentative values of u_{1k}; choose the set giving the minimum value of U.

Total-cost minimization

When reducible to 1D. Assuming initially that structures of a single type will be built in the region, the expected present value of the cost of a structure will be

$$w(c, Z) = u(c) + v(c, Z) \tag{2}$$

When earthquakes arriving to the site are idealized as a multiple Poisson process (with statistical independence between arrival times, that is, with constant hazard functions) and the demands z (or 'intensities') of base shear coefficient, corresponding to any exceedance rate $\lambda = \lambda(z)$, are defined by the z corresponding to a specific λ in every point of the region, then Z can be made equal to the intensity corresponding to a particular recurrence period. The treatment also allows variable hazard functions, but in all cases the choice of Z must be made so it can be written as $v = v(c, Z)$. Once we have zoned, any structure built in zone k, $k = 1, \ldots, K$, will be designed for a specific coefficient c_k. The cost of the structure will be $w(c_k, Z)$ and we will call it w_k. Since we are dealing with Poisson processes, the cost of all structures in zone k is

$$W_k = \iint_k \phi\, w_k\, \mathrm{d}y\, \mathrm{d}x \tag{3}$$

The cost of all structures in the region is

$$W = \sum_{k=1}^{K} W_k \tag{4}$$

The waste due to zoning is $W - W_0$, where

$$W_0 = \iint \phi\, w_0(Z)\, \mathrm{d}y\, \mathrm{d}x \tag{5}$$

$w_0(Z) = w(c_0, Z)$ and $c_0 = c_0(Z)$ is the coefficient that minimizes $w(c, Z)$ if we do not zone. As mentioned, the problem consists of minimizing W or, if preferred, of minimizing $W - W_0$. It can be split into two parts: (1) given the boundaries in each zone, compute the c_k that minimizes W_k, $k = 1, \dots, K$, and (2) find the optimum boundaries that minimize W.

The solution that we find will also be applicable to regions where two or more types $i = 1, \dots, I$, will be built and whose curves of constant Z coincide.

Optimum coefficients. If we do not zone, the optimum design coefficient would be the one that minimizes $w(c, Z)$. If w is a continuous function of c, we obtain the optimum c numerically so that w is minimum (see Appendix A, in which we explain and justify the following expressions).

Once we have defined the boundaries of zone k, the total cost is given by

$$W_k = u[(F_k - F_{k-1}) + (G_k - G_{k-1})/c_k^{\alpha_5}] \tag{6}$$

where

$$F_k = \iint \phi \, \mathrm{d}y \, \mathrm{d}x \quad \text{and} \quad G_k = \frac{\alpha_5}{\gamma} \iint \alpha_4 \, \phi \int_0^{\zeta_m} \frac{\xi(\zeta)(1 + b\xi(\zeta))}{\zeta^{\alpha_5 + 1}} \, \mathrm{d}\zeta \, \mathrm{d}y \, \mathrm{d}x$$

are integrals covering the area in which $c \leqslant c_k$. From here, we compute the optimum c_k numerically in such a way that W_k becomes minimum.

Optimality criterion: The cost at any point in zone k, $w_k = w(c_k, Z)$ and, in zone k, $W_k = \int_k \phi w_k \, \mathrm{d}A$. The coefficient c_k must be such that it minimizes W_k. If W_k is a continuous function of c_k, this coefficient is found by solving $\partial W_k/\partial c_k = 0$. Let us say that q_k denotes the position of the boundary between k and $k + 1$. When we move the boundary between two zones, we affect only c_k, c_{k+1} and the expected present value of the cost of the structures in these zones. Then, the optimum boundary is the one that minimizes

$$W_k + W_{k+1} = \int_{q_{k-1}}^{q_k} \phi w(c_k, Z) \, \mathrm{d}A + \int_{q_k}^{q_{k+1}} \phi w(c_{k+1}, Z) \, \mathrm{d}A \tag{7}$$

Thus $\partial(w_k + w_{k+1})/\partial q_k = 0$. On the other hand,

$$\partial/\partial q_k = (\partial c_k/\partial q_k)(\partial/\partial c_k) = (\partial c_{k+1}/\partial q_k)(\partial/\partial c_{k+1}) \tag{8}$$

By using Leibnitz's rule to derive an integral and employing Equation (7), we find the optimum boundary

$$\phi w(c_k, Z(x_k, y_k)) = \phi w(c_{k+1}, Z(x_k, y_k)) \tag{9}$$

This equation implies that, if ϕ is continuous on the boundary, then w must be equal on both sides of the boundary for this to be optimum [2–4].

Coincidence with curves of constant Z: The condition in Equation (9), which is translated into $w(c_k, Z(x_k, y_k)) = w(c_{k+1}, Z(x_k, y_k))$ if ϕ is the same on both sides of the boundary k, implies that Z and so w must be constant through the boundary k, $k = 1, \dots, K$. Then, every optimum boundary must coincide with curves of constant Z. In this derivative we have assumed implicitly that $\phi(x_k, y_k)$, $\partial W/\partial x$ and $\partial W/\partial y$ are continuous and that $\partial^2 W/\partial x_k^2$ and $\partial^2 W/\partial y_k^2$ are positives through the interzone boundary. If the continuity conditions are not satisfied in any curve of constant Z, then satisfying Equation (9) in a boundary coincidently with that curve is not

necessary. If a derivative of W is negative through a curve of constant Z where Equation (9) is satisfied, then this is not an optimum boundary.

Step-by-step procedure: When ϕw is a continuous function of A:

1. Estimate c_1.
2. Equating $\partial W_1/\partial c_1$ to zero, compute the values of A_1 so that c_1 is optimum.
3. Compute $w(c_1, A_1)$.
4. Compute c_2 so that $w(c_2, A_1) = w(c_1, A_1)$.
5. Repeat steps 1–5 increasing subindices in one each time until $w(c_k, A_k)$ is reached. If A_k is close enough to the value of A associated with w^+ (the maximum w in the region to be zoned) the problem is solved; otherwise, the process is repeated by starting with a new value of c.

This procedure assumes that $w(c_{k+1}, A_k) = w(c_k, A_k)$ has a single root c_{k+1}. When more roots exist, we must verify that our solution truly gives the global optimum. If ϕw is discontinuous for some values of A, we must explore the possibility that an interzone boundary coincides with those values of A.

Perturbations. We postulate first tentative boundaries in the estimated values of A_k, $k = 1, \dots, K-1$. The differences $w(c_{k+1}, A_k) - w(c_k, A_k) = \varepsilon_k$ are errors to be corrected by moving these boundaries. The required computations of the derivatives for infinitesimal changes in A_k may be cumbersome. Instead, we can introduce small finite changes ΔA_k one at a time, compute their effects on the ε_k's, assume that the ΔA's and the ε_k's are linearly related, establish $N-1$ simultaneous equations, solve them so that the ε_k's are cancelled and repeat if necessary. We assume again that $w(c_{k+1}, A_k) = w(c_k, A_k)$ has a single root c_{k+1}. When more roots exist, we must verify that our solution gives the global optimum. If ϕw is discontinuous in some values of A, we must explore the possibility that a zone boundary coincides with these values. If the Δw_k's are adequately estimated, then we converge more quickly to the exact solution than when we use the classical perturbations method. This is an additional advantage to avoid the computations of the derivatives; furthermore it allows us to use the same computer code several times.

Iterative procedure. (1) Estimate the interzone boundaries, let us say in A_k, $k = 1, \dots, K-1$
(2) Compute the corresponding optimum coefficients, c_k, $k = 1, \dots, K$.
(3) From each pair c_k, c_{k+1} compute new A_k so that result $w(c_k, A_k) = w(c_{k+1}, A_k)$, $k = 1, \dots, K-1$, is found in the boundary between these two zones.
(4) Repeat until the result is considered satisfactory.

When not reducible to 1D. Let us consider the general problem of zoning a region where structures of types $i = 1, \dots, I$ will be built. The initial cost c_i of each structure of the type depends on the vector $\mathbf{c}_i$ of its design coefficients and it can be, depending on the site, where it is erected. On the other hand, the expected present value v_i of the losses due to earthquakes in this structure is a function of c_i and of the vector $\mathbf{Z}_i$ of the parameters defining the site seismicity that are relevant to the behaviour of a structure of this type.

Let ϕ_i denote the expected present value of the number of structures of type i that will be built in the site per unit area. Let $w(c, Z) = \sum_{i=1}^{I} w_i(c_i, Z_i)\phi_i$ and $\phi = \sum_{i=1}^{I} \phi_i$, in which c and Z are the vectors of c_i and Z_i, respectively. Then, the optimality condition that we expounded for one-dimensional problems is translated into the problem that given c and Z for each boundary, the

w's are equal to both sides of the boundary. This suggests the use of methods where a boundary is postulated, then alternatively, the optimum design coefficients for each type of structure in each zone are computed, and equations allowing to calculate new boundaries between zones are obtained from them.

ZONING WITH RESTRICTIONS IN THEIR BOUNDARIES

Initial-cost minimization

When reducible to 1D. The statement of this problem is the same as the one without restrictions, but subject to the condition that the boundaries between zones coincide with well-established lines, usually jurisdictional limits whose entities we will call elementary cells.

Probably the best solution to optimize zoning in these circumstances consists of solving the problem without restrictions and then carrying the computed interzonal boundaries to the closest jurisdictional ones. Usually, this solution will be satisfactory. However, it may be advisable to improve it by exploring to which zone an elementary cell, through which the boundary without restrictions would cut, has to be assigned, inasmuch as the zone to which a cell must be assigned may depend on the assignment made to other cells cut by the unrestricted boundary. This circumstance may be detected.

When not reducible to 1D. We would now like all interzonal boundaries to coincide with portions of some predetermined lines. These lines define a number, say, P of elementary cells, such as counties. The vector of the seismicity parameters, corresponding to the most conservative design for the ith type in the Pth elementary cell, will be denoted by Z_{ip}. The problem can be visualized as the assignment of each cell to a zone. This can be done by exhaustive testing. Compared with other methods in this paper, the procedure may be practical if P is small and n is big because the number of tests is independent of n. For example with $P = 10$ and $K = 2$, the number of possibilities is 511. For big values of P it is impractical. Zoning close to the optimum can be obtained by finding optimum boundaries without restrictions and the corresponding design coefficients. Both are subsequently adjusted by assigning the elementary cells intersected by these boundaries to one zone or another according to a simple rule. For example, the whole cell may be assigned to the zone which contributes the largest value of $\int \phi u \, \mathrm{d}A$. Alternatively, we can compare the increment in U resulting in the assigning of the cell to each zone part of whose cell falls, and choose the option that causes the minimum increment. If necessary, we can perform this cell by cell and repeat the process, although it is seldom justified. The error introduced is generally insignificant compared with the cost of zoning because it is usually smaller for cells than for zones. Since we are close to the optimum, any adjustment probably produces a small increment of a higher order in U.

For problems relating to initial-cost minimization of moderate size, we can use a procedure dealing directly with the restricted boundaries. First, it is advisable to discretize the coefficients. In practice, this does not cause any lack of accuracy because it is very difficult for a code to specify base shear coefficients with more than two decimal digits. Let us say that c_i^j denotes the jth discrete value of the coefficient for the ith structural type, $i = 1, \ldots, n$, $j = 1, \ldots, n_i$ and let $c_i^j \cong Z_{ip}$ represent the smaller discrete value of c_i not smaller than Z_{ip}, $p = 1, \ldots, P$. There is no advantage in considering values that fail to satisfy $c_i^j \cong Z_{ip}$; this condition may reduce some n_i's.

Once we have chosen the c_i^j we assign n coefficients c_{ik}^j to each zone k, $k = 1, \ldots, K$, thereby forming a configuration. Such configurations must satisfy the following conditions:

1. For each i there is at least one zone k in every configuration so that $c_{ik} = \max_j c_{ik}^j$. Otherwise, some Z_{ip} would not be covered by any coefficient c_{ik}, then some elementary cells would not fit in any zone.
2. In any given configuration, there should be no zones having the same coefficients. So for any two zones k and l it is not allowed to have $c_{ik} = c_{il}$, for all i; otherwise, the two would form a single one.
3. No valid configuration can result from the zone permutation of another because the two configurations would be the same.
4. The coefficients in each elementary cell must be covered by those from at least one zone. Condition 1 is a corollary of this fourth condition.

Total-cost minimization

When reducible to 1D. The same comments made in the initial-cost minimization criterion apply here.

When not reducible to 1D. Let us say that $i = 1, \ldots, I$, $p = 1, \ldots, P$ and $k = 1, \ldots, K$, represent type of structure, elementary cell and zone, respectively. If each cell were a single zone the initial cost of structure type i at (x, y) in the cell p would be $u_i(\mathbf{c}_{ip}, x, y)$, where $\mathbf{c}_{ip}$ would be the vector of design parameters (not necessarily coefficients) for type i in this cell. The expected present value of all structures per unit area would be

$$U_i(c_{ip}, x, y) = u_i(c_{ip}, x, y)\phi_i(x, y) \tag{10}$$

Following a similar procedure as that in Appendix A the expected present value of the losses due to all earthquakes will be

$$V_i(x, y) = [\kappa L_i(c_{ip}, x, y)/\gamma] \int_0^\infty \psi(x, y, t)\, \mathrm{e}^{-\gamma t}\, \mathrm{d}t = \kappa\phi_i(x, y)\, L_i(c_{ip}, x, y)/\gamma \tag{11}$$

Combining with Equation (10) and integrating over p we will find the expected value of all types i that will be built in p if p were a single zone, $W_{ip}(\mathbf{c}_{ip}) = \iint_p [U_i(\mathbf{c}_{ip}, x, y) + V_i(\mathbf{c}_{ip}, x, y)]\, \mathrm{d}x\, \mathrm{d}y$:

$$W_{ip}(c_{ip}) = \iint_p [u_i(c_{ip}, x, y) + \kappa L_i(c_{ip}, x, y)/\gamma]\, \phi_i(x, y)\, \mathrm{d}x\, \mathrm{d}y \tag{12}$$

The following iterative procedure allows us to comply with the restrictions in the zones:

1. Estimate vectors $\mathbf{c}_{ik}$ so that they minimize $W = \sum_{k=1}^{K} \sum_i^I W_{ik}$ where W_{ik} is the sum of W_{ipk} for all p in k when structures i are designed for $\mathbf{c}_{ik}$; W_{ipk}'s are computed as W_{ip} with $\mathbf{c}_{ip}$ substituted by $\mathbf{c}_{ik}$.
2. Assign each cell p to zone k in which W_{ipk} has the smallest value. Usually, it is only necessary to try some of the k zones for each cell.
3. Compute W.
4. Compute the optimum values of $\mathbf{c}_{ik}$ for each zone.
5. Repeat steps 2–4 as necessary.

NUMERICAL EXAMPLES

Initial-cost minimization

Zoning with restrictions, two-dimensional case. Figure 2 shows a region with jurisdictional limits and isoseismals for two types of structures. We want to divide the region into two zones.

We find the values Z_{ip} and F_{ip} for each cell p that represent the non-discretized maximum required capacity for type i in cell p, respectively, and the number of structures that will be built in p. These values are displayed in Table I. For example, from Figure 2 in cell 2 we have the maximum required capacity for structures type 1 equal to 1.7 and the one corresponding to structures type 2 equal to 0.8. F_{ip} are proposed values. If we did not zone or if each cell constitute

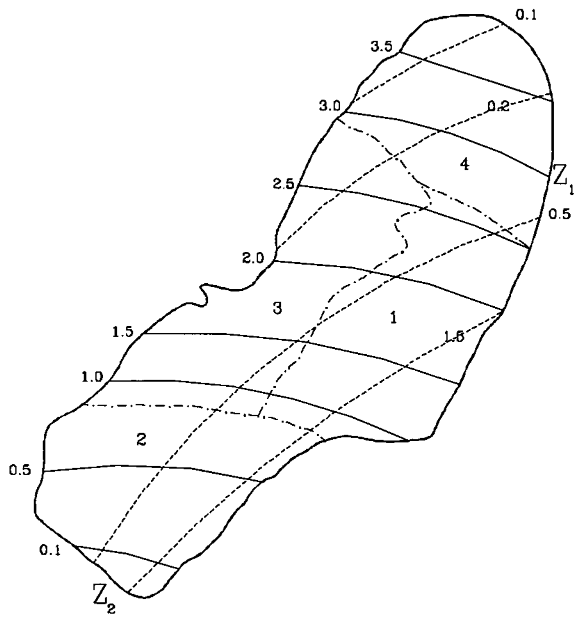

Figure 2. Jurisdictional limits and isoseismals.

Table I. Maximum required capacity and number of structures.

p	Z_{1p}	Z_{2p}	F_{1p}	F_{2p}
1	2.1	2.7	1	0
2	1.7	0.8	2	2
3	0.7	2.9	3	1
4	0.7	3.8	3	0

a zone, the total cost of the structures would be $U = \sum_{p=1}^{P} U_p$ where $U_p = \sum_{i=1}^{2} U_{ip}$. We assign possible values to c_{ik}; $i = 1,2$; $k = 1, 2$, according to the restrictions mentioned to form the configurations. For this example we choose $c_{1k} = 1, 2, 3$ and $c_{2k} = 1, 3, 4$. Neither $c_{1k} = 4$ nor $c_{2k} = 2$ are needed because there is no Z_{1p} between 3 and 4 nor Z_{2p} between 1 and 2.

Table II shows the possible number of configurations for this example. It may be observed that the following configurations are not possible: 3,4,3,4 because it would be a single zone and not two; 1,1,3,4 because with a change in notation it would be the same as that in the first row in Table II. Then we assign each cell to one of the zones. For instance, for configuration $c_{11} = 3$, $c_{21} = 4$, $c_{12} = 2$, and $c_{22} = 3$ (fifth row in Table II) we make the comparisons in each cell between

Table II. Possible configurations.

c_{11}	c_{21}	c_{12}	c_{22}
3	4	1	1
3	4	1	3
3	4	1	4
3	4	2	1
3	4	2	3
3	4	2	4
3	4	3	1
3	4	3	3
3	3	1	3
3	3	2	4

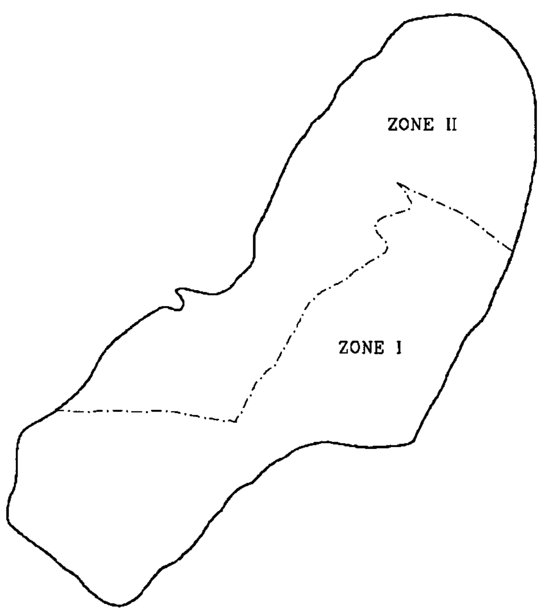

Figure 3. Zoning with restrictions, initial cost.

Z_{ip} and c_{ik} (Tables I and II respectively) as follows: If we have $p = 1$: $2.1 < 3$, $2.7 < 4$, it is assigned to zone 1 (not to 2, because $2.1 > 2$); $p = 2$: $1.7 < 2$, $0.8 < 3$ is assigned to zone 2 (not to 1 because $c_{11} > c_{12}$ *y* $c_{21} > c_{22}$); $p = 3$: $0.7 < 2$, $2.9 < 3$ is assigned to zone 2; $p = 4$: $0.7 < 3$, $3.8 < 4$ is assigned to zone 1 (not to 2 because $3.8 > 3$). Finally, we compute costs for all configurations assuming that $u_1(c) = u_2(c)$ is a linear function of c. We select the configuration with a minimum value of U. In our case, the zoning results obtained are as shown in Figure 3, with $c_{11} = 3$, $c_{12} = 1$, $c_{21} = c_{22} = 4$ (third row in Table II).

Total-cost minimization

Zoning without restrictions, perturbations method, one-dimensional case. Figure 4 shows a region of approximately 35×40 km with corresponding isoseismals of spectral ordinates for a 500-year recurrence period, and the number of structures that will be built per unit area. The region is divided into three zones.

In order to use the perturbations procedure we postulate two preliminary boundaries corresponding to coefficients $c_1 = 0.10$, $c_2 = 0.14$ and $c_3 = 0.20$. After this we establish and solve the simultaneous equations given by $w(c_{k+1}, A_k) - w(c_k, A_k) = \varepsilon_k$, with $w(\cdot) = \bar{u}(\cdot) + \bar{v}(\cdot)$ where $\bar{u}(\cdot)$ and $\bar{v}(\cdot)$ are computed as shown in Equations (A.2) and (A.12), respectively. For this example we

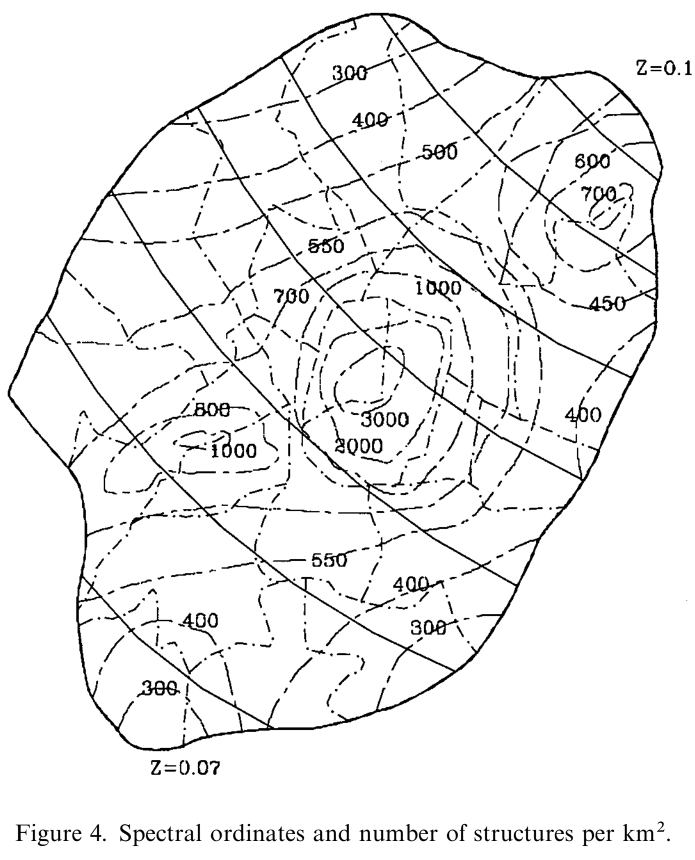

Figure 4. Spectral ordinates and number of structures per km^2.

use $\alpha_2 = 0.5$, $\alpha_3 = 1.3$, $c_0 = 0.05$, $\alpha_4 = 3.75 \times 10^{-4}$, $\alpha_5 = 3.3$, $b = 12$. We compute at this time the corresponding total cost $W = \sum_{k=1}^{3} W_k$. Where W_k is obtained as stated in Equation (A.13) (Appendix A). We do some iteration till ε_k is negligible, then finding the optimum zoning with optimum c as displayed in Table III and Figure 5. W_0 is the cost of all structures if the region is not zoned. The cost increment due to zoning is 4 per cent.

Iterative procedure, one-dimensional case. This solution gives the same results as in the perturbations method, the part of equating costs in the boundary between two zones, $w(c_{k+1}, A_k) = w(c_k, A_k)$ being the most time consuming.

Table III. Design coefficients and costs.

	Assumed values	Final values
c_1	0.10	0.11
c_2	0.14	0.16
c_3	0.20	0.195
W/C	7868.40	7687.70
$(W - W_0)/C$	476.37	295.68

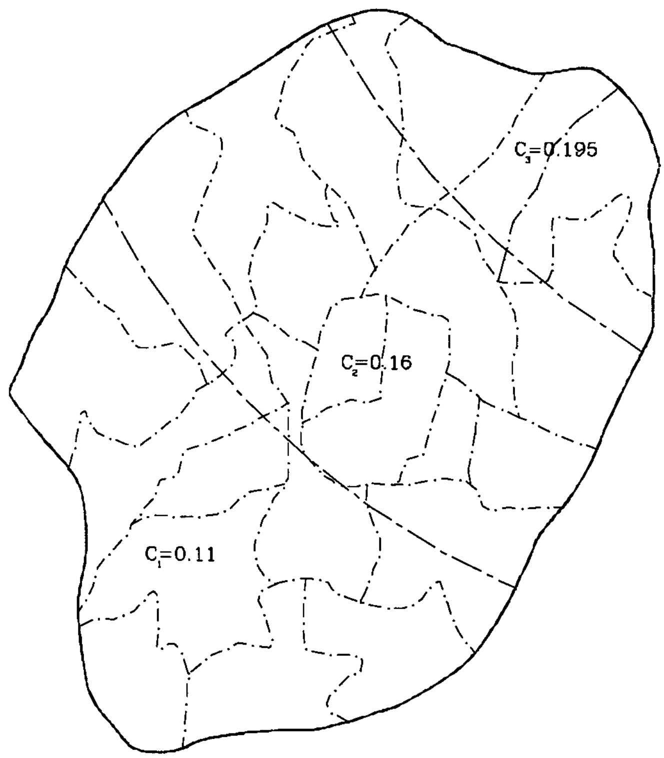

Figure 5. Zoning without restrictions, total cost.

Zoning with restrictions, two-dimensional case. Figure 6 shows a region with 16 counties and isoseismals corresponding to two types of structure. We wish to divide the region into four zones. For this example, we have $I = 2$, $K = 4$, $p = 16$ and we assume that the total cost in Equation (A.12) is given by $W_{ip} = (c_i + \bar{Z}_i/100c_i^2)\iint_p \phi_i(x, y)\,dx\,dy$. Table IV shows values $\phi_{ip}A_p$ where A_p is the area of the counties or cells, $\bar{Z}_{ip}$ is computed as $\bar{Z}_i = \int \phi_i Z_i\,dA/\int \phi_i\,dA$, c_{ip}s are the optimum coefficients without zoning in each cell computed from $c_i = (0.02\ \bar{Z}_i)^{1/3}$ also values $\bar{Z}_i\phi_i A$ of each cell are shown.

Table V shows values c_{ik}, with $i = 1, 2$, $k = 1, \ldots, 4$, estimated, based on the optimum coefficients without zoning. In the same table we show values of W_{ipk} for each cell assuming that we assign it to the zone k. For example, for cell $p = 2$ we compute $W_{ipk} = (c_{ik} + \bar{Z}_i/100c_i^2)\phi_{ip}A_p$ with $c_{1k} = c_{2k} = 0.13$, $\bar{Z}_{11} = \bar{Z}_{21} = 0.08$, we find $W_{121} = 0.58$ and $W_{221} = 0.57$, which summed up together gives 1.15. In Table V the minimum W_{ipk}'s are shown in bold type. This defines the zone where the cell under study must be assigned. It is not necessary to test each cell in each of the four zones but, only in those whose coefficients c_{ip} are close to those assigned to the zone.

We compute by iterations the minimum value of $W = \sum_{k=1}^{K}\sum_{i=1}^{I} W_{ik}$, where $W_{ik} = \sum W_{ipk}$, leads to the zoning shown in Figure 7. The optimum coefficients for each zone are computed as $c_{ik} = (0.02\ \sum_p \bar{Z}_{ip}\phi_{ip}/\sum_p f_{ip})^{1/3}$, then resulting in: $c_{11} = c_{23} = c_{24} = 0.13$, $c_{13} = c_{22} = 0.14$, $c_{12} = 0.15$, $c_{21} = 0.12$ and $c_{14} = 0.16$. The sums cover all cells of zone k.

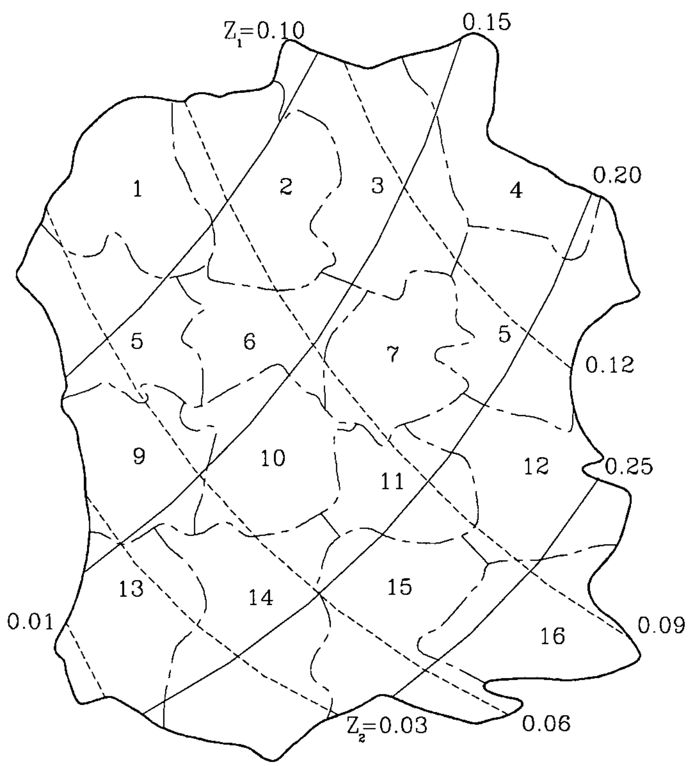

Figure 6. Jurisdictional limits and isoseismals.

Table IV. Data for the different cells of the region to be zoned.

p	$\phi_{1p}A_p$	$\phi_{2p}A_p$	$\bar{Z}_{1p}$	$\bar{Z}_{2p}$	c_{1p}	c_{2p}	$\bar{Z}_{1p}\phi_{1p}A_p$	$\bar{Z}_{2p}\phi_{2p}A_p$
1	1	0	0.08	0.08	0.12	0.12	0.08	0.00
2	3	3	0.11	0.10	0.13	0.13	0.33	0.30
3	1	2	0.14	0.11	0.14	0.13	0.14	0.22
4	8	3	0.18	0.13	0.15	0.14	1.44	0.39
5	9	1	0.10	0.07	0.13	0.11	0.90	0.07
6	3	8	0.12	0.08	0.13	0.12	0.36	0.64
7	1	1	0.16	0.10	0.15	0.13	0.16	0.10
8	4	6	0.20	0.12	0.16	0.13	0.80	0.72
9	0	9	0.13	0.05	0.14	0.10	0.00	0.45
10	1	6	0.16	0.07	0.15	0.11	0.16	0.42
11	0	7	0.19	0.08	0.16	0.12	0.00	0.56
12	5	4	0.22	0.10	0.16	0.13	1.10	0.40
13	1	6	0.18	0.02	0.15	0.07	0.18	0.12
14	8	7	0.19	0.04	0.16	0.09	1.52	0.28
15	6	9	0.22	0.07	0.16	0.11	1.32	0.63
16	9	3	0.26	0.08	0.17	0.12	2.34	0.24

Table V. Values for coefficients and costs.

k	c_{1k}	c_{2k}	1	2	3	4	5	6	7	8
1	0.13	0.13	**0.18**	**1.15**	0.60		**1.87**	**2.02**		
2	0.15	0.13	0.19		**0.60**	**2.46**			**0.41**	**2.16**
3	0.15	0.09			0.65			2.12		
4	0.16	0.11					1.96			

k	c_{1k}	c_{2k}	9	10	11	12	13	14	15	16
1	0.13	0.13								
2	0.15	0.13		1.25	1.24	2.00				
3	0.15	0.09	1.37	1.28			**0.92**	**2.85**		
4	0.16	0.11	**1.36**	**1.23**	**1.23**	**2.00**		2.88	**2.99**	**2.88**

CONCLUSIONS

In this work we have considered two design criteria to study seismic zoning. We call these criteria zonation for the initial and total cost minimization. Optimum zoning usually minimizes the expected present value of initial or total costs of the structures in a region. We developed efficient methods that optimize both unrestricted and restricted boundaries.

Initially we dealt with unrestricted boundaries and the case when under certain conditions the problem can be reduced to one dimension. Here it has been proved that when we deal with a single type of structures the problem becomes one dimensional, that the optimization criterion consists in assumption that the unit total costs of the structures built on both sides of a boundary

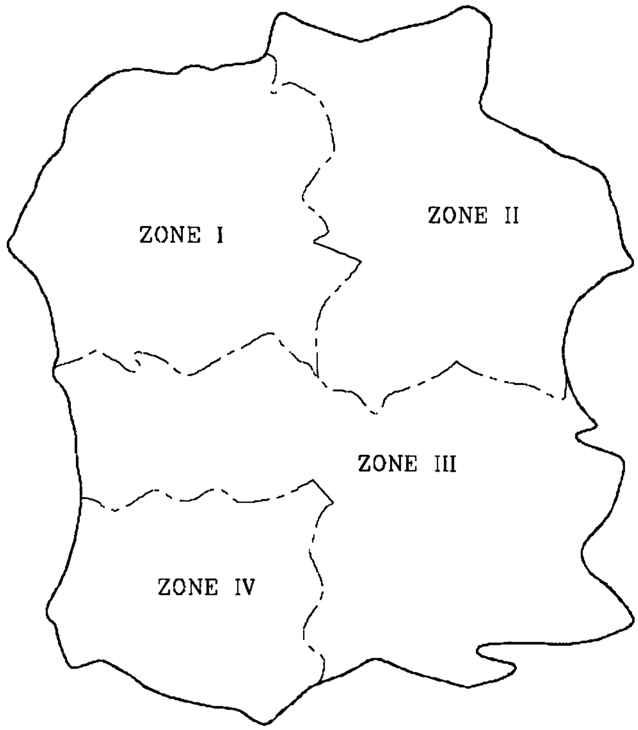

Figure 7. Zoning with restrictions.

are equal, that the boundaries coincide then with curves of constant seismicity and that the one-dimensional approach can be extended to the case in which there will be several types of structure if certain conditions are met among their unit costs and among the number of structures that will be built of each type. To solve the problem we present an iterative method and a variation of the perturbations approach.

We also deal with problems not reducible to a single dimension. Here the interzonal boundaries coincide segmentally with isoparametric curves. Exhaustive testing solves the problem accurately, but it is difficult to manage in practical cases. We developed efficient methods that optimize boundaries without restriction, and then adjust the boundaries in such a way that they meet the restrictions by giving solutions close to the optimum. A method for discretized design coefficients with restricted boundaries is presented.

The proposed methods are illustrated through examples.

APPENDIX A

We present the equations for optimum coefficients in the case where earthquakes are originated by a multiple Poisson process, without considering uncertainties neither in the attenuation laws nor in the structural properties.

The expected present value of the initial costs of structures built at (x, y) per unit area will be,

$$\bar{u} = \phi u \tag{A.1}$$

where ϕ is the expected present value of structures that will be built per unit area at (x, y), and u the initial cost of a structure designed with coefficient c. Based on studies done by Whitman *et al.* [5], Grandori [6], Ferrito [7], Rosenblueth [8], and Vargas and Jara [9] it is reasonable to adopt $u = [1 + \alpha_2 (c - c_0)^{\alpha_3}] C$ when $c \geqslant c_0$, and $u = C$ if $c \leqslant c_0$, where, if the structure is not designed against earthquakes, C would be its initial cost and c_0 would be its lateral resistance; α_2 and α_3 are constants. The expected present value of the number of structures that will be built per unit area is $\phi = \int_0^\infty \psi(t) e^{-\gamma t}\, \mathrm{d}t$, where $\psi(t)$ is the expected number of structures that will be built per unit area and per unit time, and γ the discount rate. (If $\psi(t) = \psi$ and γ are independent of time, then $\phi = \psi/\gamma$.) From this, Equation (A.1) is transformed into

$$\bar{u} = \phi [1 + \alpha_2 (c - c_0)^{\alpha_3}] C \tag{A.2}$$

Let $\kappa = \kappa(z) = -\mathrm{d}\lambda/\mathrm{d}z$ denote the density occurrence of earthquakes with intensity z, $\lambda = \lambda(z)$ the exceedance rate of z, and L_z the loss due to an earthquake of intensity z at the instant of occurrence. If we assume that the original condition is restored to the structure after each earthquake and the structure was built at instant $t = 0$, then the expected present value of the loss due to the first earthquake with intensity between z and $z + \mathrm{d}z$ is

$$v_{z1}\, \mathrm{d}z = L_z \kappa\, \mathrm{d}z \int_0^\infty \mathrm{e}^{-(\gamma + \kappa \mathrm{d}z)t}\, \mathrm{d}t = L_z \kappa\, \mathrm{d}z/(\gamma + \kappa\, \mathrm{d}z) \tag{A.3}$$

The one corresponding to the second earthquake with intensity at this interval, $v_{z2}\, \mathrm{d}z$, will be $v_{z1}\, \mathrm{d}z$ times $\kappa\, \mathrm{d}z/(\gamma + \kappa\, \mathrm{d}z)$, and so on. Thus the contribution of all earthquakes with intensity in $(z, z + \mathrm{d}z)$ will be

$$\mathrm{d}v_z = L_z \sum_{n=1}^{\infty} [\kappa\, \mathrm{d}z/(\gamma + \kappa\, \mathrm{d}z)]^n\, \mathrm{d}z = L_z (\kappa/\gamma)\, \mathrm{d}z \tag{A.4}$$

(See Reference [10]). This shows that the expected present value of the losses due to all earthquakes in a structure built at $t = 0$ is

$$v = (1/\gamma) \int_0^{z_m} L_z \kappa\, \mathrm{d}z \tag{A.5}$$

where z_m is the maximum intensity that can occur at the site of interest.

If the structure had been built at instant $t \geqslant 0$, this would also be the expectance of the value updated to time t, of the losses caused by all earthquakes that will occur in ulterior times. Since the number of structures per unit area $\psi(t)\, \mathrm{d}t$ will be built between instants t and $t + \mathrm{d}t$, the expected present value of the losses per unit area will be

$$\bar{v} = \int_0^\infty v \psi \mathrm{e}^{-\gamma t}\, \mathrm{d}t = (\phi/\gamma) \int_0^{z_m} L_z \kappa\, \mathrm{d}z \tag{A.6}$$

We will take L_z formed by two terms, the first representing direct material damages suffered by the building itself before the strike of an earthquake of intensity z. We will write this term in the form $L_z = u\xi(z, c)$. The function ξ must increase with z, thereby decreasing as c increases so that $\lim_{z \to 0} \xi = 0$ and $\lim_{z \to \infty} \xi = 1$. Furthermore, it must tend very fast to zero when z tends to zero because we know that earthquakes of low intensity do not cause any damage. The second term

represents the other damages (indirect economics and not economics) that earthquakes cause to society: It must be insignificant when ξ is small because then the content of buildings does not suffer practically any damage. It must tend to a higher quantity than the first term when this is approximated to one because then we are dealing with buildings that suffer collapse, usually causing almost total loss of its content, loss of many human lives and commotion of the economy in the affected area. Considering this, we will take $L_z = u\xi(z,c)[1 + b\xi(z, c)]$ where b is a factor much greater than one. According to data and analyses made by Esteva *et al.* [11] and Ordaz *et al.* [12, 13], given an earthquake characterized by z, the expected value of the loss due to physical damage to the building itself at the time of the earthquake is proportional to power 1.6 of the quotient $\zeta = z/c$ of the intensity to the design coefficient in the interval $1 \leqslant \zeta \leqslant 7$. According to empirical data and considerations made the following expressions are used for $\xi(z, c) = \xi(\zeta)$: $\xi(\zeta) = 0.025\zeta^6 - 0.015\zeta^9$ if $\zeta \leqslant 1$ and $\xi(\zeta) = (0.188 + \zeta^{1.8})/(117.8 + \zeta^{1.8})$ if $\zeta > 1$. Substituting in Equation (A.6) we obtain

$$\bar{v} = \frac{\phi u}{\gamma} \int_0^{z_m} \xi(z/c)(1 + b\xi(z/c)\kappa(z)\,\mathrm{d}z \tag{A.7}$$

According to Cornell and Vanmarcke [14], we will take the exceedance rate of the magnitudes of earthquakes originated in a tectonic province as

$$\lambda(M) = \alpha_1(\mathrm{e}^{-\beta M} - \mathrm{e}^{-\beta M_m}) \tag{A.8}$$

where M means magnitude, M_m is the maximum value of M that can be generated in the province, and α_1 and β are constants. On the other hand, most of the formulas for attenuation laws give the peak ground acceleration, velocity and displacement, and the response spectra ordinates for both a given period and degree of damping for big distances from the origin, as z equal to a function of the focal coordinates and those of the site of interest multiplied by $\exp(\beta' M)$ where β' is a constant. Combining this expression with Equation (A.8) we obtain

$$\lambda(z) = \alpha_4(z^{-\alpha_5} - z_m^{-\alpha_5}) \tag{A.9}$$

where α_4 and α_5 are constants. The expression for λ is valid when crust material behaves linearly between the source and the site of interest, and the distance between this and the source is big compared with the dimensions of the rupture area. Then we can write

$$\kappa(z) = \alpha_4\alpha_5 z^{-\alpha_5 - 1} \tag{A.10}$$

substituting in (A.7) gives

$$\bar{v} = \frac{\alpha_4\alpha_5\phi u}{\gamma} \int_0^{z_m} \frac{\xi(z/c)(1 + b\xi(z/c))}{z^{\alpha_5 + 1}}\,\mathrm{d}z \tag{A.11}$$

which with the change of variable $\zeta = z/c$ is converted to

$$\bar{v} = \frac{\alpha_4\alpha_5\phi u}{\gamma c^{\alpha_5}} \int_0^{\zeta_m} \frac{\xi(\zeta)(1 + b\xi(\zeta))}{\zeta^{\alpha_5 + 1}}\,\mathrm{d}\zeta \tag{A.12}$$

where $\zeta_m = z_m/c$.

If we did not zone, the optimum design coefficient would be the one that minimizes the total cost given by the sum of Equations (A.2) and (A.12). The optimum value of c can be computed numerically so it minimizes w.

Once the boundaries for zone k are defined, the total cost is given by

$$W_k = u[(F_k - F_{k-1}) + (G_k - G_{k-1})/c_k^{\alpha_5}] \tag{A.13}$$

where $F_k = \iint \phi \, dy \, dx$ and

$$G_k = \frac{\alpha_5}{\gamma} \iint \alpha_4 \phi \int_0^{\zeta_m} \frac{\xi(\zeta)(1 + b\xi(\zeta))}{\zeta^{\alpha_5 + 1}} \, d\zeta \, dy \, dx$$

these double integrals cover the area in that $c \leqslant c_k$. In the same manner as before, the optimum c_k is computed numerically so that W_k is minimum. The form of Equation (A.13) does not change essentially when either other functional forms of the exceedance rates or non-poissonian arrivals of earthquakes are considered when we recognize the uncertainties mentioned.

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